(b) Estimate the number of cubic feet of natural gas produced by Iran in 2008.

(c) Using the model in part (b), predict when Iran's natural gas production reaches 4.2 trillion cubic feet.

50. *Natural Gas Production*

(a) Find a natural logarithm regression equation for the data in Table 1.17 and superimpose its graph on a scatter plot of the data. Let $x = 0$ represent 2000.

TABLE 1.17 China's Natural Gas Production

Year	Cubic Feet (trillions)
2002	1.15
2003	1.21
2004	1.44
2005	1.76
2006	2.07

Source: Statistical Abstract of the United States, 2010.

(b) Estimate the number of cubic feet of natural gas produced by China in 2008.

(c) Using the model in part (b), predict when China's natural gas production reaches 2.25 trillion cubic feet.

51. Group Activity *Inverse Functions* Let $y = f(x) = mx + b$, $m \neq 0$.

(a) **Writing to Learn** Give a convincing argument that f is a one-to-one function.

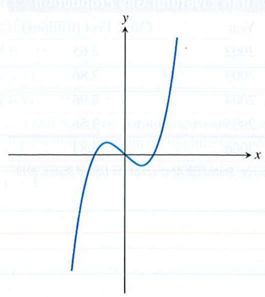
(b) Find a formula for the inverse of f. How are the slopes of f and f^{-1} related?

(c) If the graphs of two functions are parallel lines with a nonzero slope, what can you say about the graphs of the inverses of the functions?

(d) If the graphs of two functions are perpendicular lines with a nonzero slope, what can you say about the graphs of the inverses of the functions?

Standardized Test Questions

52. True or False The function displayed in the graph below is one-to-one. Justify your answer.



53. True or False If $(f \circ g)(x) = x$, then g is the inverse function of f. Justify your answer.

In Exercises 54 and 55, use the function $f(x) = 3 - \ln(x + 2)$.

54. Multiple Choice Which of the following is the domain of f?

(A) $x \neq -2$ (B) $(-\infty, \infty)$ (C) $(-2, \infty)$
(D) $[-1.9, \infty)$ (E) $(0, \infty)$

55. Multiple Choice Which of the following is the range of f?

(A) $(-\infty, \infty)$ (B) $(-\infty, 0)$ (C) $(-2, \infty)$
(D) $(0, \infty)$ (E) $(0, 5.3)$

56. Multiple Choice Which of the following is the inverse of $f(x) = 3x - 2$?

(A) $g(x) = \dfrac{1}{3x - 2}$ (B) $g(x) = x$ (C) $g(x) = 3x - 2$
(D) $g(x) = \dfrac{x - 2}{3}$ (E) $g(x) = \dfrac{x + 2}{3}$

57. Multiple Choice Which of the following is a solution of the equation $2 - 3^{-x} = -1$?

(A) $x = -2$ (B) $x = -1$ (C) $x = 0$
(D) $x = 1$ (E) There are no solutions.

Exploration

58. *Supporting the Quotient Rule* Let $y_1 = \ln(x/a)$, $y_2 = \ln x$, $y_3 = y_2 - y_1$, and $y_4 = e^{y_3}$.

(a) Graph y_1 and y_2 for $a = 2, 3, 4$, and 5. How are the graphs of y_1 and y_2 related?

(b) Graph y_3 for $a = 2, 3, 4$, and 5. Describe the graphs.

(c) Graph y_4 for $a = 2, 3, 4$, and 5. Compare the graphs to the graph of $y = a$.

(d) Use $e^{y_3} = e^{y_2 - y_1} = a$ to solve for y_1.

Extending the Ideas

59. *One-to-One Functions* If f is a one-to-one function, prove that $g(x) = -f(x)$ is also one-to-one.

60. *One-to-One Functions* If f is a one-to-one function and $f(x)$ is never zero, prove that $g(x) = 1/f(x)$ is also one-to-one.

61. *Domain and Range* Suppose that $a \neq 0$, $b \neq 1$, and $b > 0$. Determine the domain and range of the function.

(a) $y = a(b^{c-x}) + d$ (b) $y = a \log_b (x - c) + d$

62. Group Activity *Inverse Functions*

Let $f(x) = \dfrac{ax + b}{cx + d}$, $c \neq 0$, $ad - bc \neq 0$.

(a) **Writing to Learn** Give a convincing argument that f is one-to-one.

(b) Find a formula for the inverse of f.

(c) Find the horizontal and vertical asymptotes of f.

(d) Find the horizontal and vertical asymptotes of f^{-1}. How are they related to those of f?

1.6 Trigonometric Functions

What you will learn about . . .

- Radian Measure
- Graphs of Trigonometric Functions
- Periodicity
- Even and Odd Trigonometric Functions
- Transformations of Trigonometric Graphs
- Inverse Trigonometric Functions

and why . . .

Trigonometric functions can be used to model periodic behavior and applications such as musical notes.

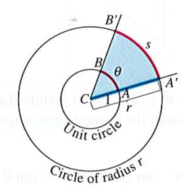


Figure 1.40 The radian measure of angle ACB is the length θ of arc AB on the unit circle centered at C. The value of θ can be found from any other circle, however, as the ratio s/r.

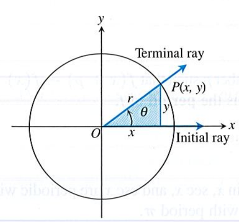


Figure 1.41 An angle θ in standard position.

Radian Measure

The **radian measure** of the angle ACB at the center of the unit circle (Figure 1.40) equals the length of the arc that ACB cuts from the unit circle.

EXAMPLE 1 Finding Arc Length

Find the length of an arc subtended on a circle of radius 3 by a central angle of measure $2\pi/3$.

SOLUTION

According to Figure 1.40, if s is the length of the arc, then

$$s = r\theta = 3(2\pi/3) = 2\pi.$$

Now Try Exercise 1.

When an angle of measure θ is placed in *standard position* at the center of a circle of radius r (Figure 1.41), the six basic trigonometric functions of θ are defined as follows:

$$\textbf{sine: } \sin\theta = \frac{y}{r} \qquad \textbf{cosecant: } \csc\theta = \frac{r}{y}$$

$$\textbf{cosine: } \cos\theta = \frac{x}{r} \qquad \textbf{secant: } \sec\theta = \frac{r}{x}$$

$$\textbf{tangent: } \tan\theta = \frac{y}{x} \qquad \textbf{cotangent: } \cot\theta = \frac{x}{y}$$

Graphs of Trigonometric Functions

When we graph trigonometric functions in the coordinate plane, we usually denote the independent variable (radians) by x instead of θ. Figure 1.42 on the next page shows sketches of the six trigonometric functions. It is a good exercise for you to compare these with what you see in a grapher viewing window. (Some graphers have a "trig viewing window.")

EXPLORATION 1 Unwrapping Trigonometric Functions

Set your grapher in *radian mode*, parametric mode, and *simultaneous mode* (all three). Enter the parametric equations

$$x_1 = \cos t, \quad y_1 = \sin t \quad \text{and} \quad x_2 = t, \quad y_2 = \sin t.$$

1. Graph for $0 \le t \le 2\pi$ in the window $[-1.5, 2\pi]$ by $[-2.5, 2.5]$. Describe the two curves. (You may wish to make the viewing window square.)
2. Use TRACE to compare the y-values of the two curves.
3. Repeat part 2 in the window $[-1.5, 4\pi]$ by $[-5, 5]$, using the parameter interval $0 \le t \le 4\pi$.
4. Let $y_2 = \cos t$. Use TRACE to compare the x-values of curve 1 (the unit circle) with the y-values of curve 2 using the parameter intervals $[0, 2\pi]$ and $[0, 4\pi]$.
5. Set $y_2 = \tan t$, $\csc t$, $\sec t$, and $\cot t$. Graph each in the window $[-1.5, 2\pi]$ by $[-2.5, 2.5]$ using the interval $0 \le t \le 2\pi$. How is a y-value of curve 2 related to the corresponding point on curve 1? (Use TRACE to explore the curves.)

Angle Convention: Use Radians

From now on in this book it is assumed that all angles are measured in radians unless degrees or some other unit is stated explicitly. When we talk about the angle $\pi/3$, we mean $\pi/3$ radians (which is 60°), not $\pi/3$ degrees. When you do calculus, keep your calculator in radian mode.

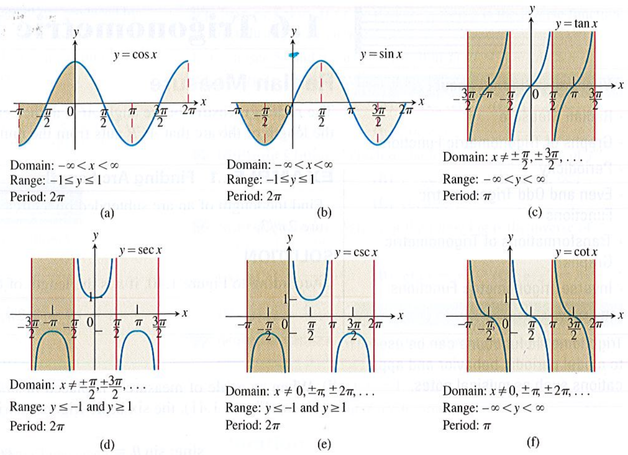


Figure 1.42 Graphs of the (a) cosine, (b) sine, (c) tangent, (d) secant, (e) cosecant, and (f) cotangent functions using radian measure.

Periods of Trigonometric Functions

Period π: $\tan(x + \pi) = \tan x$
$\cot(x + \pi) = \cot x$

Period 2π: $\sin(x + 2\pi) = \sin x$
$\cos(x + 2\pi) = \cos x$
$\sec(x + 2\pi) = \sec x$
$\csc(x + 2\pi) = \csc x$

Periodicity

When an angle of measure θ and an angle of measure $\theta + 2\pi$ are in standard position, their terminal rays coincide. The two angles therefore have the same trigonometric function values:

$$\begin{aligned}\cos(\theta + 2\pi) &= \cos\theta \quad & \sin(\theta + 2\pi) &= \sin\theta \quad & \tan(\theta + 2\pi) &= \tan\theta \\ \sec(\theta + 2\pi) &= \sec\theta \quad & \csc(\theta + 2\pi) &= \csc\theta \quad & \cot(\theta + 2\pi) &= \cot\theta\end{aligned} \tag{1}$$

Similarly, $\cos(\theta - 2\pi) = \cos\theta$, $\sin(\theta - 2\pi) = \sin\theta$, and so on.

We see the values of the trigonometric functions repeat at regular intervals. We describe this behavior by saying that the six basic trigonometric functions are *periodic*.

DEFINITION Periodic Function, Period

A function $f(x)$ is **periodic** if there is a positive number p such that $f(x + p) = f(x)$ for every value of x. The smallest such value of p is the **period** of f.

As we can see in Figure 1.42, the functions $\cos x$, $\sin x$, $\sec x$, and $\csc x$ are periodic with period 2π. The functions $\tan x$ and $\cot x$ are periodic with period π.

Even and Odd Trigonometric Functions

The graphs in Figure 1.42 suggest that $\cos x$ and $\sec x$ are even functions because their graphs are symmetric about the y-axis. The other four basic trigonometric functions are odd.

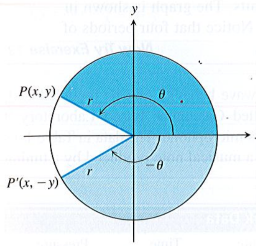


Figure 1.43 Angles of opposite sign. (Example 2)

EXAMPLE 2 Confirming Even and Odd

Show that cosine is an even function and sine is odd.

SOLUTION

From Figure 1.43 it follows that

$$\cos(-\theta) = \frac{x}{r} = \cos\theta, \quad \sin(-\theta) = \frac{-y}{r} = -\sin\theta,$$

so cosine is an even function and sine is odd.

Now Try Exercise 5.

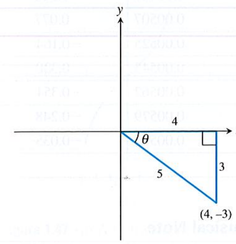


Figure 1.44 The angle θ in standard position. (Example 3)

EXAMPLE 3 Finding Trigonometric Values

Find all the trigonometric values of θ if $\sin\theta = -3/5$ and $\tan\theta < 0$.

SOLUTION

The angle θ is in the fourth quadrant, as shown in Figure 1.44, because its sine and tangent are negative. From this figure we can read that $\cos\theta = 4/5$, $\tan\theta = -3/4$, $\csc\theta = -5/3$, $\sec\theta = 5/4$, and $\cot\theta = -4/3$.

Now Try Exercise 9.

Transformations of Trigonometric Graphs

The rules for shifting, stretching, shrinking, and reflecting the graph of a function apply to the trigonometric functions. The following diagram will remind you of the controlling parameters.

Vertical stretch or shrink; reflection about x-axis
Vertical shift

$$y = af(b(x + c)) + d$$

Horizontal stretch or shrink; reflection about y-axis
Horizontal shift

The general sine function or **sinusoid** can be written in the form

$$f(x) = A\sin\left[\frac{2\pi}{B}(x - C)\right] + D,$$

where $|A|$ is the *amplitude*, $|B|$ is the *period*, C is the *horizontal shift*, and D is the *vertical shift*.

EXAMPLE 4 Graphing a Trigonometric Function

Determine the **(a)** period, **(b)** domain, **(c)** range, and **(d)** draw the graph of the function $y = 3\cos(2x - \pi) + 1$.

SOLUTION

We can rewrite the function in the form

$$y = 3\cos\left[2\left(x - \frac{\pi}{2}\right)\right] + 1.$$

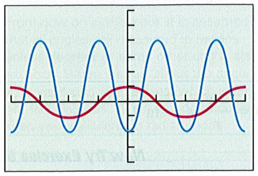
(a) The period is given by $2\pi/B$, where $2\pi/B = 2$. The period is π.

(b) The domain is $(-\infty, \infty)$.

(c) The graph is a basic cosine curve with amplitude 3 that has been shifted up 1 unit. Thus, the range is $[-2, 4]$.

continued

$y = 3\cos(2x - \pi) + 1$, $y = \cos x$



$[-2\pi, 2\pi]$ by $[-4, 6]$

Figure 1.45 The graph of $y = 3\cos(2x - \pi) + 1$ (blue) and the graph of $y = \cos x$ (red). (Example 4)

(d) The graph has been shifted to the right $\pi/2$ units. The graph is shown in Figure 1.45 together with the graph of $y = \cos x$. Notice that four periods of $y = 3\cos(2x - \pi) + 1$ are drawn in this window. ***Now Try Exercise 13.***

Musical notes are pressure waves in the air. The wave behavior can be modeled with great accuracy by general sine curves. Devices called Calculator Based Laboratory™ (CBL) systems can record these waves with the aid of a microphone. The data in Table 1.18 give pressure displacement versus time in seconds of a musical note produced by a tuning fork and recorded with a CBL system.

TABLE 1.18 Tuning Fork Data

Time	Pressure	Time	Pressure	Time	Pressure
0.00091	−0.080	0.00271	−0.141	0.00453	0.749
0.00108	0.200	0.00289	−0.309	0.00471	0.581
0.00125	0.480	0.00307	−0.348	0.00489	0.346
0.00144	0.693	0.00325	−0.248	0.00507	0.077
0.00162	0.816	0.00344	−0.041	0.00525	−0.164
0.00180	0.844	0.00362	0.217	0.00543	−0.320
0.00198	0.771	0.00379	0.480	0.00562	−0.354
0.00216	0.603	0.00398	0.681	0.00579	−0.248
0.00234	0.368	0.00416	0.810	0.00598	−0.035
0.00253	0.099	0.00435	0.827		

EXAMPLE 5 Finding the Frequency of a Musical Note

Consider the tuning fork data in Table 1.18.

(a) Find a sinusoidal regression equation (general sine curve) for the data and superimpose its graph on a scatter plot of the data.

(b) The *frequency* of a musical note, or wave, is measured in cycles per second, or hertz (1 Hz = 1 cycle per second). The frequency is the reciprocal of the *period* of the wave, which is measured in seconds per cycle. Estimate the frequency of the note produced by the tuning fork.

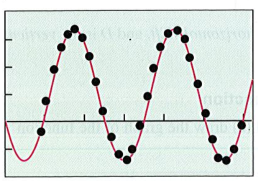
SOLUTION

(a) The sinusoidal regression equation produced by our calculator is approximately

$$y = 0.6\sin(2488.6x - 2.832) + 0.266.$$

Figure 1.46 shows its graph together with a scatter plot of the tuning fork data.

$y = 0.6\sin(2488.6x - 2.832) + 0.266$



$[0, 0.0062]$ by $[-0.5, 1]$

Figure 1.46 A sinusoidal regression model for the tuning fork data in Table 1.18. (Example 5)

(b) The period is $\dfrac{2\pi}{2488.6}$ sec, so the frequency is $\dfrac{2488.6}{2\pi} \approx 396$ Hz.

Interpretation The tuning fork is vibrating at a frequency of about 396 Hz. On the pure tone scale, this is the note G above middle C. It is a few cycles per second different from the frequency of the G we hear on a piano's tempered scale, 392 Hz. ***Now Try Exercise 23.***

Inverse Trigonometric Functions

None of the six basic trigonometric functions graphed in Figure 1.42 is one-to-one. These functions do not have inverses. However, in each case the domain can be restricted to produce a new function that does have an inverse, as illustrated in Example 6.

EXAMPLE 6 Restricting the Domain of the Sine

Show that the function $y = \sin x$, $-\pi/2 \le x \le \pi/2$, is one-to-one, and graph its inverse.

SOLUTION

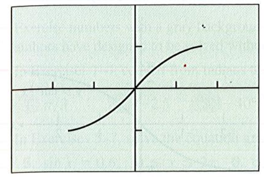
Figure 1.47a shows the graph of this restricted sine function using the parametric equations

$$x_1 = t, \quad y_1 = \sin t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

This restricted sine function is one-to-one because it does not repeat any output values. It therefore has an inverse, which we graph in Figure 1.47b by interchanging the ordered pairs using the parametric equations

$$x_2 = \sin t, \quad y_2 = t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$ ***Now Try Exercise 25.***

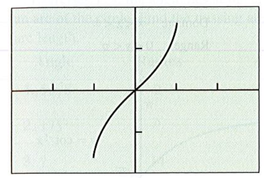
$x = t$, $y = \sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$



$[-3, 3]$ by $[-2, 2]$

(a)

$x = \sin t$, $y = t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$



$[-3, 3]$ by $[-2, 2]$

(b)

Figure 1.47 (a) A restricted sine function and (b) its inverse. (Example 6)

The inverse of the restricted sine function of Example 6 is called the *inverse sine function*. The inverse sine of x is the angle whose sine is x. It is denoted by $\sin^{-1} x$ or arcsin x. Either notation is read "arcsine of x" or "the inverse sine of x."

The domains of the other basic trigonometric functions can also be restricted to produce a function with an inverse. The domains and ranges of the resulting inverse functions become parts of their definitions.

DEFINITIONS Inverse Trigonometric Functions

Function	Domain	Range
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \sec^{-1} x$	$\lvert x \rvert \ge 1$	$0 \le y \le \pi, y \ne \frac{\pi}{2}$
$y = \csc^{-1} x$	$\lvert x \rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$
$y = \cot^{-1} x$	$-\infty < x < \infty$	$0 < y < \pi$

The graphs of the six inverse trigonometric functions are shown in Figure 1.48.

EXAMPLE 7 Finding Angles in Degrees and Radians

Find the measure of $\cos^{-1}(-0.5)$ in degrees and radians.

SOLUTION

Put the calculator in degree mode and enter $\cos^{-1}(-0.5)$. The calculator returns 120, which means 120 degrees. Now put the calculator in radian mode and enter $\cos^{-1}(-0.5)$. The calculator returns 2.094395102, which is the measure of the angle in radians. You can check that $2\pi/3 \approx 2.094395102$. ***Now Try Exercise 27.***

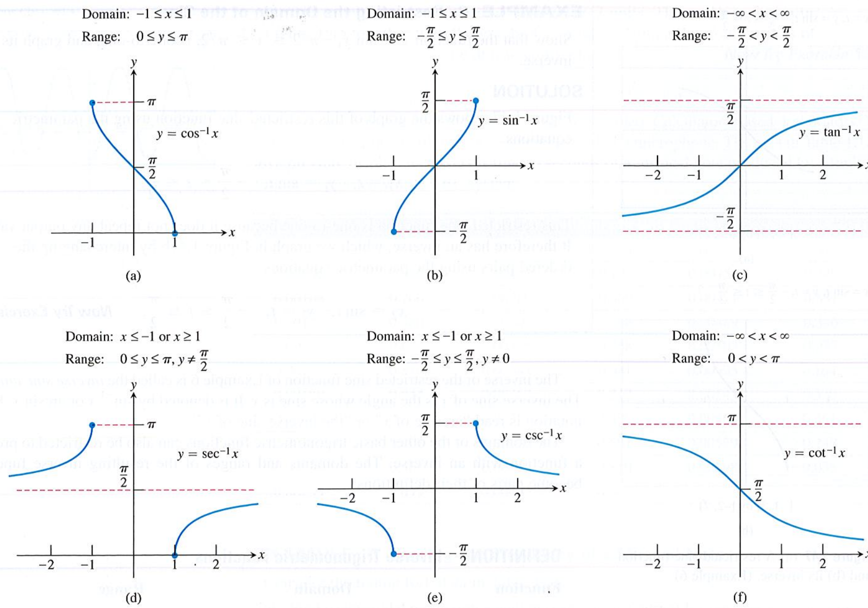


Figure 1.48 Graphs of (a) $y = \cos^{-1} x$, (b) $y = \sin^{-1} x$, (c) $y = \tan^{-1} x$, (d) $y = \sec^{-1} x$, (e) $y = \csc^{-1} x$, and (f) $y = \cot^{-1} x$.

EXAMPLE 8 Using the Inverse Trigonometric Functions

Solve for x.

(a) $\sin x = 0.7$ in $0 \le x < 2\pi$

(b) $\tan x = -2$ in $-\infty < x < \infty$

SOLUTION

(a) Notice that $x = \sin^{-1}(0.7) \approx 0.775$ is in the first quadrant, so 0.775 is one solution of this equation. The angle $\pi - x$ is in the second quadrant and has sine equal to 0.7. Thus two solutions in this interval are

$$\sin^{-1}(0.7) \approx 0.775 \quad \text{and} \quad \pi - \sin^{-1}(0.7) \approx 2.366.$$

(b) The angle $x = \tan^{-1}(2) \approx -1.107$ is in the fourth quadrant and is the only solution to this equation in the interval $-\pi/2 < x < \pi/2$ where $\tan x$ is one-to-one. Since $\tan x$ is periodic with period π, the solutions to this equation are of the form

$$\tan^{-1}(-2) + k\pi \approx -1.107 + k\pi$$

where k is any integer.

Now Try Exercise 31.

Quick Review 1.6 *(For help, go to Sections 1.2 and 1.6.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved without a calculator.

In Exercises 1–4, convert from radians to degrees or degrees to radians.

1. $\pi/3$ **2.** -2.5 **3.** $-40°$ **4.** $45°$

In Exercises 5–7, solve the equation graphically in the given interval.

5. $\sin x = 0.6, \quad 0 \le x \le 2\pi$ **6.** $\cos x = -0.4, \quad 0 \le x \le 2\pi$

7. $\tan x = 1, \quad -\frac{\pi}{2} \le x < \frac{3\pi}{2}$

8. Show that $f(x) = 2x^2 - 3$ is an even function. Explain why its graph is symmetric about the y-axis.

9. Show that $f(x) = x^3 - 3x$ is an odd function. Explain why its graph is symmetric about the origin.

10. Give one way to restrict the domain of the function $f(x) = x^4 - 2$ to make the resulting function one-to-one.

Section 1.6 Exercises

In Exercises 1–4, the angle lies at the center of a circle and subtends an arc of the circle. Find the missing angle measure, circle radius, or arc length.

Angle	Radius	Arc Length
1. $5\pi/8$	2	?
2. $175°$	?	10
3. ?	14	7
4. ?	6	$3\pi/2$

In Exercises 5–8, determine if the function is even or odd.

5. secant **6.** tangent

7. cosecant **8.** cotangent

In Exercises 9 and 10, find all the trigonometric values of θ with the given conditions.

9. $\cos\theta = -\frac{15}{17}, \quad \sin\theta > 0$

10. $\tan\theta = -1, \quad \sin\theta < 0$

In Exercises 11–14, determine **(a)** the period, **(b)** the domain, **(c)** the range, and **(d)** draw the graph of the function.

11. $y = 3\csc(3x + \pi) - 2$ **12.** $y = 2\sin(4x + \pi) + 3$

13. $y = -3\tan(3x + \pi) + 2$

14. $y = 2\sin\left(2x + \frac{\pi}{3}\right)$

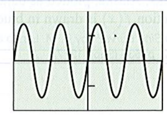
In Exercises 15 and 16, choose an appropriate viewing window to display two complete periods of each trigonometric function in radian mode.

15. **(a)** $y = \sec x$ **(b)** $y = \csc x$ **(c)** $y = \cot x$

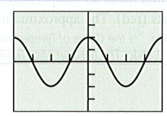
16. **(a)** $y = \sin x$ **(b)** $y = \cos x$ **(c)** $y = \tan x$

In Exercises 17–22, specify **(a)** the period, **(b)** the amplitude, and **(c)** identify the viewing window that is shown.

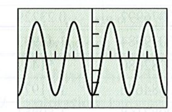
17. $y = 1.5\sin 2x$



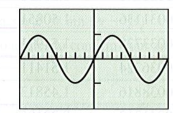
18. $y = 2\cos 3x$



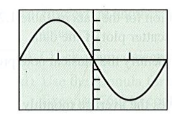
19. $y = -3\cos 2x$



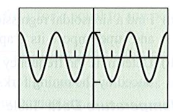
20. $y = 5\sin\frac{x}{2}$



21. $y = -4\sin\frac{\pi}{3}x$



22. $y = \cos\pi x$



23. Group Activity A musical note like that produced with a tuning fork or pitch meter is a pressure wave. Table 1.19 gives frequencies (in Hz) of musical notes on the tempered scale. The pressure versus time tuning fork data in Table 1.20 were collected using a CBL™ and a microphone.

TABLE 1.19 Frequencies of Notes

Note	Frequency (Hz)
C	262
$C^\#$ or D^b	277
D	294
$D^\#$ or E^b	311
E	330
F	349
$F^\#$ or G^b	370
G	392
$G^\#$ or A^b	415
A	440
$A^\#$ or B^b	466
B	494
C (next octave)	524

Source: CBL™ System Experimental Workbook, Texas Instruments, Inc., 1994.

TABLE 1.20 Tuning Fork Data

Time (s)	Pressure	Time (s)	Pressure
0.0002368	1.29021	0.0049024	−1.06632
0.0005664	1.50851	0.0051520	0.09235
0.0008256	1.51971	0.0054112	1.44694
0.0010752	1.51411	0.0056608	1.51411
0.0013344	1.47493	0.0059200	1.51971
0.0015840	0.45619	0.0061696	1.51411
0.0018432	−0.89280	0.0064288	1.43015
0.0020928	−1.51412	0.0066784	0.19871
0.0023520	−1.15588	0.0069408	−1.06072
0.0026016	−0.04758	0.0071904	−1.51412
0.0028640	1.36858	0.0074496	−0.97116
0.0031136	1.50851	0.0076992	0.23229
0.0033728	1.51971	0.0079584	1.46933
0.0036224	1.51411	0.0082080	1.51411
0.0038816	1.45813	0.0084672	1.51971
0.0041312	0.32185	0.0087168	1.50851
0.0043904	−0.97676	0.0089792	1.36298
0.0046400	−1.51971		

(a) Find a sinusoidal regression equation for the data in Table 1.20 and superimpose its graph on a scatter plot of the data.

(b) Determine the frequency of and identify the musical note produced by the tuning fork.

24. *Temperature Data* Table 1.21 gives the average monthly temperatures for St. Louis for a 12-month period starting with January. Model the monthly temperature with an equation of the form

$$y = a \sin [b(t - h)] + k,$$

y in degrees Fahrenheit, t in months, as follows:

TABLE 1.21 Temperature Data for St. Louis

Time (months)	Temperature (°F)
1	34
2	30
3	39
4	44
5	58
6	67
7	78
8	80
9	72
10	63
11	51
12	40

(a) Find the value of b assuming that the period is 12 months.

(b) How is the amplitude a related to the difference $80° - 30°$?

(c) Use the information in (b) to find k.

(d) Find h, and write an equation for y.

(e) Superimpose a graph of y on a scatter plot of the data.

In Exercises 25–26, show that the function is one-to-one, and graph its inverse.

25. $y = \cos x, \quad 0 \le x \le \pi$ **26.** $y = \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

In Exercises 27–30, give the measure of the angle in radians and degrees. Give exact answers whenever possible.

27. $\sin^{-1}(0.5)$ **28.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

29. $\tan^{-1}(-5)$ **30.** $\cos^{-1}(0.7)$

In Exercises 31–36, solve the equation in the specified interval.

31. $\tan x = 2.5, \quad 0 \le x \le 2\pi$

32. $\cos x = -0.7, \quad 2\pi \le x < 4\pi$

33. $\csc x = 2, \quad 0 < x < 2\pi$ **34.** $\sec x = -3, \quad -\pi \le x < \pi$

35. $\sin x = -0.5, \quad -\infty < x < \infty$

36. $\cot x = -1, \quad -\infty < x < \infty$

In Exercises 37–40, use the given information to find the values of the six trigonometric functions at the angle θ. Give exact answers.

37. $\theta = \sin^{-1}\left(\frac{8}{17}\right)$ **38.** $\theta = \tan^{-1}\left(-\frac{5}{12}\right)$

39. The point $P(-3, 4)$ is on the terminal side of θ.

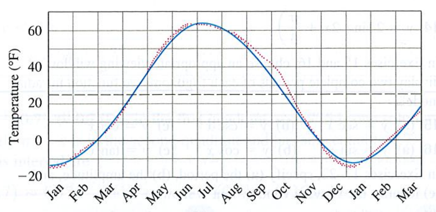
40. The point $P(-2, 2)$ is on the terminal side of θ.

In Exercises 41 and 42, evaluate the expression.

41. $\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)$

42. $\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right)$

43. *Temperatures in Fairbanks, Alaska* Find the **(a)** amplitude, **(b)** period, **(c)** horizontal shift, and **(d)** vertical shift of the model used in the figure below. **(e)** Then write the equation for the model.



Normal mean air temperature for Fairbanks, Alaska, plotted as data points (red). The approximating sine function $f(x)$ is drawn in blue.

Source: "Is the Curve of Temperature Variation a Sine Curve?" by B. M. Lando and C. A. Lando, The Mathematics Teacher, 7.6, Fig. 2, p. 535 (Sept. 1977).

44. *Temperatures in Fairbanks, Alaska* Use the equation of Exercise 43 to approximate the answers to the following questions about the temperatures in Fairbanks, Alaska, shown in the figure in Exercise 43. Assume that the year has 365 days.

(a) What are the highest and lowest mean daily temperatures?

(b) What is the average of the highest and lowest mean daily temperatures? Why is this average the vertical shift of the function?

45. *Even-Odd*

(a) Show that $\cot x$ is an odd function of x.

(b) Show that the quotient of an even function and an odd function is an odd function.

46. *Even-Odd*

(a) Show that $\csc x$ is an odd function of x.

(b) Show that the reciprocal of an odd function is odd.

47. *Even-Odd* Show that the product of an even function and an odd function is an odd function.

48. *Finding the Period* Give a convincing argument that the period of $\tan x$ is π.

49. *Sinusoidal Regression* Table 1.22 gives the values of the function

$$f(x) = a \sin (bx + c) + d$$

accurate to two decimals.

TABLE 1.22 Values of a Function

x	$f(x)$
1	3.42
2	0.73
3	0.12
4	2.16
5	4.97
6	5.97

(a) Find a sinusoidal regression equation for the data.

(b) Rewrite the equation with a, b, c, and d rounded to the nearest integer.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

50. True or False The period of $y = \sin(x/2)$ is π. Justify your answer.

51. True or False The amplitude of $y = \frac{1}{2}\cos x$ is 1. Justify your answer.

In Exercises 52–54, $f(x) = 2\cos(4x + \pi) - 1$.

52. Multiple Choice Which of the following is the domain of f?

(A) $[-\pi, \pi]$ (B) $[-3, 1]$ (C) $[-1, 4]$ (D) $(-\infty, \infty)$ (E) $x \ne 0$

53. Multiple Choice Which of the following is the range of f?

(A) $(-3, 1)$ (B) $[-3, 1]$ (C) $(-1, 4)$ (D) $[-1, 4]$ (E) $(-\infty, \infty)$

54. Multiple Choice Which of the following is the period of f?

(A) 4π (B) 3π (C) 2π (D) π (E) $\pi/2$

55. Multiple Choice Which of the following is the measure of $\tan^{-1}(-\sqrt{3})$ in degrees?

(A) $-60°$ (B) $-30°$ (C) $30°$ (D) $60°$ (E) $120°$

Exploration

56. *Trigonometric Identities* Let $f(x) = \sin x + \cos x$.

(a) Graph $y = f(x)$. Describe the graph.

(b) Use the graph to identify the amplitude, period, horizontal shift, and vertical shift.

(c) Use the formula

$$\sin \alpha \cos \beta + \cos \alpha \sin \beta = \sin(\alpha + \beta)$$

for the sine of the sum of two angles to confirm your answers.

Extending the Ideas

57. Exploration Let $y = \sin(ax) + \cos(ax)$.

Use the symbolic manipulator of a computer algebra system (CAS) to help you with the following:

(a) Express y as a sinusoid for $a = 2, 3, 4$, and 5.

(b) Conjecture another formula for y for a equal to any positive integer n.

(c) Check your conjecture with a CAS.

(d) Use the formula for the sine of the sum of two angles (see Exercise 56c) to confirm your conjecture.

58. Exploration Let $y = a \sin x + b \cos x$.

Use the symbolic manipulator of a computer algebra system (CAS) to help you with the following:

(a) Express y as a sinusoid for the following pairs of values: $a = 2, b = 1$; $a = 1, b = 2$; $a = 5, b = 2$; $a = 2, b = 5$; $a = 3, b = 4$.

(b) Conjecture another formula for y for any pair of positive integers. Try other values if necessary.

(c) Check your conjecture with a CAS.

(d) Use the following formulas for the sine or cosine of a sum or difference of two angles to confirm your conjecture.

$$\sin \alpha \cos \beta \pm \cos \alpha \sin \beta = \sin(\alpha \pm \beta)$$

$$\cos \alpha \cos \beta \pm \sin \alpha \sin \beta = \cos(\alpha \mp \beta)$$

In Exercises 59 and 60, show that the function is periodic and find its period.

59. $y = \sin^3 x$ **60.** $y = |\tan x|$

In Exercises 61 and 62, graph one period of the function.

61. $f(x) = \sin(60x)$ **62.** $f(x) = \cos(60\pi x)$